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The DOGEV Model

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THE DOGEV MODEL

Abstract

At present, there appears to be no qualitative dependent model that can simultaneously account for data sets in which the variable of interest is potentially ordered but also has strong heterogeneity of the observed outcomes. This heterogeneity of particular outcomes, inherently attracts individuals to them, in addition to that determined by the individual's observed characteristics. An example of such unobserved heterogeneity would be brand-loyalty (or "captivity") in a model of consumer choice. Such heterogeneity of the outcomes, may well result in a pronounced multi-modal distribution of the variable of interest. This paper introduces the Dogit Ordered Generalized Extreme Value (DOGEV) model, which does account for both ordering and captivity (and/or multiple modes) in the data. In the spirit of Manski (1977), the DOGEV model combines a model for choice set generation with the Ordered Generalized Extreme Value model. We illustrate the model using three different empirical examples: a model of employment contract types; an inflationary expectations data set and; a survey of students' evaluations of teaching. These three examples are chosen as they represent different values that the additional ancillary parameters are likely to take in practice.

Keywords: Generalized extreme value, choice set generation, ordinal data.
JEL Classification: C25

1 Introduction

The use of qualitative response discrete choice models is now widespread in the fields of economics, transportation research and marketing, to name a few. Predominantly this takes the form of analyzing unit data, often responses to survey questionnaires. A distinguishing feature of many such data sets, is that some (or all) of the outcomes of the variable of interest, have a certain inherent attractiveness, over and above that determined by the individual's observed characteristics. This could be termed heterogeneity of the *outcome*.

Examples of such heterogeneity include numerical responses to particular question of the type "how much do you think x will change over the next year?". Here the researcher is likely to witness the presence of *digit preferencing* - certain numbers, such as 0, 5 and 10 tend to be favored. An example from the marketing literature, would be in a model of consumer brand choice. Here, in addition to relative prices, income *etc.*, there may well be "brand-loyalty" which additionally pulls individuals towards particular outcomes. A further example can be taken from the discrete labor supply field where, presumably for institutional reasons, workers are likely to be restricted to the discretized labour supply hours corresponding to full-time, part-time and non-work points. That is, although workers may prefer to work "unusual" hours, these options are simply not open to them.

In essence, it is heterogeneity of the outcome(s) that is additionally attracting individuals towards these. If the variable of interest strongly embodies such captivity (or digit preferencing, or brand-loyalty), this could well be evidenced by a multi-modal distribution of observed outcomes (although this is not necessarily the case). It is also the case, that in many modeling instances, the variable of interest is also potentially ordered. Indeed, with all of the above examples, as well as with many other ones, there is likely to be ordering in the observed responses.

Traditionally the researcher would tackle a discrete choice modeling exercise using the multinomial logit (MNL) model. The MNL model is computationally tractable but does embody strong behavioral assumptions. Research has, therefore, focussed either upon computationally intensive estimation methods that allow for the estimation of more flexible discrete choice models, such as the multinomial probit or random parameters logit models, or upon developing computationally tractable and flexible functional forms for discrete choice models. The latter has stemmed from McFadden's (McFadden 1978) *Generalized Extreme Value* (GEV) class of models. Indeed, notwithstanding the huge advances in simulation based estimation, there has recently been an upsurge of interest in deriving new models that are members of the GEV class (see, for example, Breshanan, Stern, and Trajtenberg 1997, Vovsha 1998, Koppelman and Wen 2000, Swait 2001, Wen and Koppelman 2001).

On the other hand, when a researcher is faced with a discrete choice modeling exercise where there are concerns about potential ordering in the data, it is highly likely that ordered probit or logit models would be applied. These models do account for the ordinal nature of the outcome but are not consistent with the models derived from the random utility framework such as the multinomial probit or GEV class of models. Of particular interest to this paper is the one extension has been proposed to the GEV class of models by Small (1987) and Small (1994) to account for outcomes that are implicitly ordered such that a stochastic correlation between choices of close proximity is introduced into the model - the *Ordered Generalized Extreme Value* (OGEV) model.

However, neither of these approaches specifically accounts for the situation described above, where there is captivity in the variable of interest (which may be evidenced by clustering of observed responses around several alternatives *c.f.* digit preferencing). A model that can account for this is the DOGIT model (Gaudry and Dagenais 1979). In spite of its apparent applicability in numerous situations, the DOGIT model has seldom been applied in practice (for examples, see Gaudry and Wills 1979, Gaudry 1980, Tse 1987, Bordley 1990, Kannan and Yim 2001, Chandrasekharan, McCarthy, and Wright 1994). The DOGIT model contains additional parameters to the standard MNL model, which are most easily interpreted as "captivity", "loyalty", "gravity" or "preference" coefficients. That is, the DOGIT models allows individuals to be captive to particular choices, for example if the outcomes themselves possess qualities that make them more attractive in some sense. Indeed, it is this particular aspect of the DOGIT model that makes it so appropriate for multi-modal data such as digit preferencing where individuals tend to gravitate to integers in multiples of 5 and 10.

Whilst the DOGIT model is able to cope with the potential captivity and multiple modes in discrete choice data, a potential drawback is that, as with the MNL model, it does not allow for any ordering in the data.[1]

[1] McFadden (1981) argues that a deficiency of the DOGIT model is its lack of consistency with the standard RUM framework. However, as we show in this paper, the DOGIT model is consistent with the broader framework of Manski (1977)

In this paper we utilize the framework derived by Manski (1977) that ties together the choice set generation process with the outcome selection process to introduce a new model, the Dogit Ordered Generalized Extreme Value (DOGEV) model. The DOGEV model combines the flexibility of the random utility maximization model that allows coefficients to vary across outcomes and the ordering and correlation of proximate choice properties of the OGEV model, with the choice set generation process and implied captivity properties of the DOGIT model. This yields a specification that is able to model discrete choice data that potentially exhibits captivity in the choice outcomes (and/or is multi-modal) *and* ordinal in nature. It also allows for simple tests of both ordering and captivity.

To illustrate the DOGEV model we apply it to three different applications. These are chosen because they are interesting in their own right, and also illustrate three likely scenarios a practitioner is likely to encounter. In the DOGEV model the (additional) captivity parameters, to ensure a proper probability density function, must be strictly greater or equal to zero. However, in practice some (or all) of these may be freely estimated to non-zero; estimated to be zero or restricted *a priori* to be zero. The first application is based upon Brown, Farrell, Harris, and Sessions's (2002) model, which considers the ordering of employment-types according to their associated income risk. Here, all of the captivity parameters are freely estimated to be non-zero.

The second application is based on Australian inflationary expectations' data. From a policy perspective, it is important to understand what drives inflationary expectations. Such expectations have far reaching implications for the macroeconomy as a whole, especially in an environment of an active inflationary setting policy regime. Inflationary expectations are likely to affect the processes of wage bargaining, price setting and asset allocation, amongst other things, and as a consequence are likely to direct affect monetary policy and the activities of central banks. Again, this data is pertinent for our purposes, as the numerical answers would appear to be ordered, and there is clear evidence of digit preferencing at inflationary expectations of 0, 5 and 10%. In this application, all captivity parameters are estimated, and some go to their boundary solutions of zero.

Finally, we consider a model of student evaluations of satisfaction with the quality of a large first-year undergraduate compulsory course in Statistics taught in a business faculty in an Australian University. Since such data may be used in the tenure and promotion process understanding the determinants of successful outcomes is important. Such evaluation data is ordinal in nature and, potentially, certain responses (*e.g.* strongly agree, strongly disagree and neutral) may exercise a gravitational pull. Here, for *a priori* reasons, we restrict some captivity parameters to be zero.

The plan of this paper is as follows. In Section 2 the DOGEV model is derived. Sections 3, 4 and 5, respectively contain the applications to: employment contract types, inflationary expectations and student evaluations. Section 6 concludes.

2 A New Ordered Probability Model

As is common in the literature (Fry, Brooks, Comley, and Zhang 1993), we start with the standard random utility maximization (RUM) model, with indirect utility function given by

$$U_{ij} = V_{ij} + \varepsilon_{ij}, \tag{1}$$

with $i = 1, \ldots N$, $j = 1, \ldots J$. U_{ij} is the utility individual i gains from alternative j, which is typically assumed to be a (linear in parameters) function of observed individual characteristics $\mathbf{x}_i$ and of the characteristics of the outcomes $\mathbf{z}_{ij}$, such that $V_{ij} = \mathbf{x}_i'\beta_j + \mathbf{z}_{ij}'\alpha$. Finally, ε_{ij} is a random disturbance term assumed to follow an independent Extreme Value distribution. The individual is assumed to choose the outcome that maximizes utility. With choice set $C = \{1, \ldots, J\}$,the associated MNL probabilities are given by

$$P_{ij}^{MNL} = \frac{\exp(V_{ij})}{\sum_{k=1}^{J} \exp(V_{ik})}, \tag{2}$$

where to identify the model restrictions on the model parameters are required (Maddala 1983).

2.1 The DOGIT Model

MNL models are popular in practice, primarily because of their simplicity and ease of estimation. However, this simplicity does impose some strong restrictions on the model, most notably that of the *Independence of Irrelevant Alternatives* (IIA). The IIA property of MNL models, essentially says that the *odds ratio*, P_{ij}/P_{ik}, $j \neq k$, is independent of all other alternatives, and independent of additions to, and deletions from, the full choice set. In many instances this appears to be an unrealistic assumption, a problem exacerbated by the fact that tests for IIA generally have very poor power properties (Fry and Harris 1996).

A number of non-IIA alternatives to the MNL have been proposed. The DOGIT model of Gaudry and Dagenais (1979) appears attractive for many modelling instances, as its probabilities expand on the MNL ones of equation (2) so avoiding the IIA property. Specifically, the DOGIT probabilities are given by

$$P_{ij}^{DOGIT} = \frac{\exp(V_{ij}) + \theta_j \sum_{k=1}^{J} \exp(V_{ik})}{\left(1 + \sum_{k=1}^{J} \theta_k\right) \sum_{k=1}^{J} \exp(V_{ik})}. \tag{3}$$

A requirement of the DOGIT model is that, to ensure a proper probability density function, the θ parameters, are non-negative, $\theta_j \geq 0 \; \forall \; j = 1, \ldots, J$.

Consistent with the approach in Manski (1977), the DOGIT model can also be conceptualized as arising from a two part choice process (Fry and Harris 1996). Manski showed that the discrete choice problem comprises of two components: a choice set generation process and (conditional on choice set selection) an outcome selection process. In particular

$$P_{ij} = \sum_{C \subset B_i} P_i(j \mid C) P_i(C)$$

where B_i is the set of all non-empty choice sets available to individual i, $P_i(j \mid C)$ is the probability that individual i chooses outcome j given that the choice set is C and $P_i(C)$ is the probability that individual i selects choice set C. The number of choice sets available to an individual can in theory be very large. Thus researchers typically place some restrictions on the choice set generation process. In the first component of the model we determine the choice set selection probabilities, $P_i(C)$ and in the second component we determine the conditional outcome selection probabilities, $P_i(j \mid C)$.

For the DOGIT model an individual is assumed either "captive" to one of the J outcomes or chooses from the full choice set. Therefore, the available choice set faced by the individual, $B_i = B \, \forall \, i$, comprises $J+1$ sets, J single outcome "captivity sets" and one set comprising all J outcomes from which "free choice" is (subsequently) exercised by the individual. The choice set generation process itself can be represented as a random utility maximization model with utilities given by

$$U_{ik}^{1} = W_{ik} + \epsilon_{ik}, \;\; i = 1, \ldots, n; \;\; k = 1, \ldots, J+1. \tag{4}$$

Under the assumptions that ϵ_{ik} are independent identically distributed Extreme Value, that $W_{ik} = \log(\theta_k)$ and the normalization that $W_{iJ+1} = 0$, the probability of individual i choosing a single outcome (captive) choice set is given by

$$P_{ij} = \frac{\theta_j}{1 + \sum_{k=1}^{J} \theta_k}, \tag{5}$$

and the probability that individual i chooses the full choice set is

$$P_{iJ+1} = \frac{1}{1 + \sum_{k=1}^{J} \theta_k}. \tag{6}$$

For the outcome selection process the probability that an individual chooses the specified outcome j from a single outcome choice set is one and the probability that an individual chooses the specified outcome j from the full choice set is given by the standard RUM model that leads to the MNL in (2) above. Thus, utilizing the Manski framework, the DOGIT model is given by

$$P_{ij}^{DOGIT} = \frac{\theta_j}{1+\sum_{k=1}^{J}\theta_k} + \frac{1}{1+\sum_{k=1}^{J}\theta_k} \times P_{ij}^{MNL}. \tag{7}$$

The parameterization of equation (7) illustrates a further boundary condition on the admissible range for the θ_j values (in addition to $\theta_j \geq 0 \ \forall \ j = 1, \ldots, J$). In the limit, the proportion choosing outcome j in a sample, must be greater or equal to the proportion given by the "captive" probability $P_{ij}^{captive}$, where

$$P_{ij}^{captive} = \frac{\theta_j}{1+\sum_{k=1}^{J}\theta_k}.$$

Effectively, this places an upper bound on the admissible θ_j values.

In such a parameterization, the θ's can be interpreted as "preference", "loyalty" or "gravity" parameters or alternatively heterogeneity of the outcome(s). Of course, it is possible to generalize the DOGIT model further by allowing these gravity parameters to be a function of observed heterogeneity, indeed, this is the parameterized Logit captivity model of Swait and Ben-Akiva (1987). However, this is not considered in this paper, as the model is already deemed to be sufficiently heavily parameterized. As we do not parameterize the preference parameters but treat them as fixed constants, they can be thought of as representing unobserved heterogeneity of the outcome, the strength of which can (and is almost certain to), vary across j, but be constant across i.

At one extreme, if the pull of these gravity parameters is "large" for any particular outcome they are likely to dominate the ultimate choice probabilities for that outcome - irrespective of observed personal heterogeneity. At the other extreme, a zero θ value for an outcome results in choice probabilities being driven solely by observed heterogeneity. In between these extremes, choice probabilities are a combination of the two. An example with regard to modeling inflationary expectations, might be that if headline inflation has been at 2% for several periods, $\theta_{j=2}$ may be such that the probability of choosing 2% is predominantly unaffected by individuals' characteristics which otherwise one would have expected to be influential - individuals are simply drawn to this outcome.

2.2 The OGEV Model

Small (1987) introduces a discrete choice model, the Ordered Generalized Extreme Value (OGEV), that is a member of the Generalized Extreme Value class of models. Again, the OGEV probabilities expand on the MNL ones of equation (2) such that IIA is no longer embodied. However, the underlying motivation for the OGEV model is to provide a suitable model for outcomes that are ordered in some sense, whilst still providing the flexibility of the MNL model, as compared to that of the ordered probit model, for example. Unlike the MNL or DOGIT probabilities, the OGEV ones embody a correlation between outcomes in close proximity. Such a correlation appears likely for ordered data in many instances, especially where the observed outcomes are realizations of an underlying latent scale. For example, given a five-point response scale ($j = 1, \ldots, 5$) say, of satisfaction, individuals may choose "neutral" ($j = 3$), but be heavily influenced by the neighboring choices of "moderately satisfied" ($j = 4$) and "moderately dissatisfied" ($j = 2$).

Although it is possible to allow the window of correlation to be arbitrarily large, this increases the number of parameters to be estimated and makes estimation cumbersome (Small 1987). Therefore we restrict attention to the standard OGEV model (Small 1987). In Small's notation we have $M = 2$ and $\rho_r = \rho$ for all r. However, for some applications (such as our inflationary expectations example), it is possible that the more flexible correlation structure implied by multiple ρ might be more appropriate. For reasons of parsimony we do not consider this model variant.

The standard OGEV model implies a correlation between outcomes that are near neighbors. Analogously to an moving average process, this correlation decreases the further away two outcomes j and k are, and moreover is zero when $|j-k|>2$. Although they cannot be written explicitly in closed form (Small 1987), these correlations are *inversely* related to the parameter ρ. The standard OGEV probabilities are given by

$$
\begin{aligned}
P_{ij}^{OGEV} &= \frac{\exp\left(\rho^{-1}V_{ij}\right)}{\sum_{r=1}^{J+1}\left(\exp\left(\rho^{-1}V_{ir-1}\right)+\exp\left(\rho^{-1}V_{ir}\right)\right)^{\rho}} \qquad (8)\\
&\times \Big[\left(\exp\left(\rho^{-1}V_{ij-1}\right)+\exp\left(\rho^{-1}V_{ij}\right)\right)^{\rho-1} \\
&+ \left(\exp\left(\rho^{-1}V_{ij}\right)+\exp\left(\rho^{-1}V_{ij+1}\right)\right)^{\rho-1}\Big],
\end{aligned}
$$

with the convention that $\exp(\rho^{-1}V_{i0}) = \exp(\rho^{-1}V_{iJ+1}) = 0$ and $0 < \rho \leq 1$.

As $\rho \to 1$, OGEV probabilities converge to MNL ones, which gives a simple parameter restriction ($\rho = 1$) based test of the OGEV versus MNL formulations. Such a test is also implicitly a test of ordering versus non-ordering of the outcomes in the choice set. Finally, we note that as $\rho \to 0$, the associated cumulative distribution function is a degenerate one, but one still consistent with random utility maximization (Small 1987).

2.3 The DOGEV Model

The OGEV model does not allow for the phenomenon of captivity and/or multiple modes and the DOGIT model does not allow either for the ordering of outcomes or for the (potential) correlation of proximate outcomes. In this section we combine the ideas underlying both the DOGIT and OGEV models to produce a new discrete choice model, the Dogit Ordered Generalized Extreme Value (DOGEV) model.

The idea is to combine the two part choice generating process of the DOGIT model with the proximate correlation and ordering of the OGEV model. Specifically, we assume that the choice set generation process leading to captive (or digit preference or brand-loyalty), probabilities is as in the DOGIT formulation above. That is, we allow the outcomes themselves to possess unobserved heterogeneity/characteristics which potentially attract individuals to them. This attraction is in addition to any the outcomes already possess, as determined by the individual and their observed characteristics.

In the second component of the choice process, if free choice is exercised, the selection probabilities are given by the OGEV formulation. Thus, using equations (7) and (8) the selection probabilities for the DOGEV model are given by

$$P_{ij}^{DOGEV} = \frac{\theta_j}{1 + \sum_{k=1}^{M} \theta_k} + \frac{1}{1 + \sum_{k=1}^{M} \theta_k} \times P_{ij}^{OGEV}. \tag{9}$$

The nested models are therefore

$$
\begin{array}{ll}
\text{OGEV:} & \theta_1 = \ldots = \theta_M = 0,\ 0 < \rho \leq 1, \\
\text{DOGIT:} & \rho = 1, \text{ at least one } \theta_j > 0,\ j = 1, \ldots, J, \\
\text{MNL:} & \theta_1 = \ldots = \theta_M = 0,\ \rho = 1,
\end{array}
$$

of which the last two do not imply ordering in the observed outcomes. This model is flexible whilst allowing one to model captive (and/or multi-modal) ordinal data in a simple way. Models are nested within DOGEV in a way that allows for hypothesis testing based model selection. Thus the DOGEV model is not restricted to applications where clustering of observed responses (multiple modes) and ordering is a potential data issue, but through its nested variants (DOGIT, OGEV and MNL) may be used in any application where the distribution of observed outcomes is either captive or multi-modal or ordered or unordered.

The parameters of the DOGEV model can be consistently estimated using the maximum likelihood criterion. By defining an indicator variable d_{ij} as

$$
d_{ij} = \begin{cases} 1 & \text{if individual } i \text{ chooses alternative } j \\ 0 & \text{otherwise} \end{cases}
$$

the log-likelihood function will be

$$
\ell^m (\phi \,|X) = \sum_{j=1}^{J} \sum_{i=1}^{N} d_{ij} \ln P_{ij}^m, \tag{10}
$$

with m =MNL, DOGIT, OGEV and DOGEV and with $\phi' = \left[\text{vec}\beta_j, \alpha\right]$, $\left[\left(\text{vec}\beta_j, \alpha\right)', \theta'\right]$, $\left[\left(\text{vec}\beta_j, \alpha\right)', \rho\right]$ and $\left[\left(\text{vec}\beta_j, \alpha\right)', \theta', \rho\right]$, respectively.

In practice, it is possible that some, all, or none of the captivity parameters may go their boundary solutions. Moreover, it is also possible to set some of these captivity parameters to their lower boundary solutions for *a priori* reasons. The following applications illustrate such scenarios. Note that $\theta_k = 0$, simply implies no captivity for choice k.

3 Application 1: Employment Contracts

In a recent paper Brown, Farrell, Harris, and Sessions (2002) consider the degree of 'income risk' associated with the three broad types of employment contract. Salaried employment, for example, implies relatively stable pay. In contrast, a self-employed worker can offer increasingly competitive tenders as the demand for his/her labor services declines, thereby maintaining a vestige of employment, albeit at or near his/her reservation wage. For such individuals price is fluid. PRP contracts, comprising an element of both fixed and variable pay, offer a middle road between these two extremes. That is, Brown, Farrell, Harris, and Sessions (2002) attempt to explain an individual's employment contract-type, whilst recognizing that these outcomes are necessarily ordered in terms of their associated income risk - the presumption is that self-employment is relatively more risky than PRP, which is itself relatively more risky than salaried employment.

If individuals were identical in terms of their attitudes towards risk, and in the absence of ability and/or capital constraints, one would anticipate a pooling equilibrium with all workers flocking to one of the three contacts. More realistically, a spectrum of risk aversion and the presence of such constraints would imply a separating equilibrium, with the expected utility of employment across each of the three contracts types being equalized.[2] Following Brown and Sessions (2002), a spectrum of contracts is assumed, which can be nested in the illustrative form

$$w_j = (1 - \lambda_j)\,\bar{w} + \lambda_j f\,(e, \zeta) \tag{11}$$

[2]Here we focus on a simplified model, and do not consider the relationship between individuals' risk preference/aversion and their choice of employment contract-type.

where: $j = s, prp$ and se (denoting 'salaried', 'PRP' and 'self-employment', respectively); w_j denotes total remuneration; $\bar{w}$ the component of total remuneration that is independent of worker performance; $f(e, \zeta)$ some function mapping the relationship between worker performance, ζ, and a stochastic parameter, e and; λ_j the proportion of total remuneration that is dependent upon performance. It is presumed that $\lambda_s = 0$, $\lambda_{prp} \in \{0, 1\}$ and $\lambda_{se} = 1$, such that $w_s = \bar{w}$, $w_{prp} = (1 - \lambda_{prp}) \bar{w} + \lambda_{prp} f(e, \zeta)$ and $w_{se} = f(e, \zeta)$. For simplicity assume that ζ can take two values - "high" H, and "low" L, $\zeta = (\zeta_H, \zeta_L)$, with $f(e, \zeta_H) > f(e, \zeta_L)$, the range of income to which an individual is exposed is defined by $\Delta w = \lambda_j [f(e, \zeta_H) - f(e, \zeta_L)]$, with the implication that income risk is increasing in λ_j.

A similar dataset, but simplified specification, is used as in Brown, Farrell, Harris, and Sessions (2002). The data comes from the Family Expenditure Survey (FES), which is a nationally representative survey conducted in the United Kingdom. Some 10,000 households are selected each year to take part, and the average response rate is approximately 70%. The main aim of the survey is to provide a reliable source of information on household expenditure, income and other aspects of household finances. It contains detailed information on employment contracts, individual specific characteristics and household specific characteristics. The sub-sample comprises of working adults aged between 18 and 65 who are either self-employed or salaried or a contract characterized by a bonus scheme (bonus). Pooling the annual years 1997-1998, 1998-1999 and 1999-2000, gives us a large working sample of over thirteen and a half thousand individuals.

3.1 Results

The DOGEV estimation results are presented in Table 1. Although focus is not on the structural part of the model, it appears to be relatively well specified, with region, education, occupational class, industry, age, housing tenure-type and number of children, all appearing to strongly affect the choice of employment contract-type.

Turning to the ancillary parameters, all of the θ_j's are significantly different from zero (recalling that these are one-sided tests, due to the constraint that $\theta_j \geq 0 \ \forall \ j = 1, \ldots, J$). It appears that the extent of captivity is strongest for salaried workers, then PRP workers and finally, the self-employed. This could well be due to institutional constraints or demand side (employer) effects (for example, more salaried positions are available than PRP ones, than self-employment ones). It may also represent the fact that individuals are inherently risk averse and drawn to the reduced income risk associated with salaried contracts.

In terms of ρ, the parameter is strongly significantly different from zero. However, as $\rho \rightarrow 1$, the DOGEV$\rightarrow$MNL, and the extent of correlation is therefore inversely related to ρ, being zero when $\rho = 1$. Therefore, there is only weak evidence of ordering in employment contact-types. However, recalling that this parameterization embodies the *standard* OGEV model, which implies a correlation between near neighbors only, it might be the case that it is difficult to precisely estimate ρ when J is "small" (here $J = 3$), akin to effective sample size arguments and the MNL estimation of β.

3.2 Some Model Evaluations

Due to the complexity of such a model, it is unclear from the estimated coefficients how well the model describes the data, and moreover what the implications for the data are. Here we present the results from a brief exercise which attempts to do this. In Figure 1 we present sample proportions of observed choices (as a reference point), along with the implied extent of DOGEV "preference" for each contract-type. In addition we also present the total predicted probabilities of the DOGEV model, evaluated at the sample means of the explanatory variables.

As can be seen, the DOGEV model appears to fit the data well, closely replicating the observed sample proportions. The relative size of the estimated θ parameters is translated into the extent of captivity for each choice. Therefore, we witness that regardless of observed personal characteristics, the estimated probability that an (employed) individual will choose a salaried contract, is 30%. Once one additionally includes (average) personal characteristics, this over doubles to just under 70%. Total probabilities for an individual choosing, or being in, a PRP contract, are significantly lower, but again about half of this is driven by individuals being captive to this choice. In total, the typical individual has a 7% chance of being self-employed, of which 2 percentage points arise from captivity, with the remainder being driven by observed characteristics and the associated OGEV probabilities.

In summary, allowing for ordering and captivity/preferencing in the data appears to yield sensible estimates. Moreover, in terms of predicted probabilities for a "typical" individual compared to sample proportions, the DOGEV model appears to fit the data well, in addition to having a sensible interpretation.

4 Application 2: Inflationary Expectations

Inflationary expectations have wide reaching implications for the economy as a whole - especially so in a policy regime of active inflation targeting. For example, the Reserve Bank of Australia (RBA) is currently committed to an annual rate of inflation in the range of 2 - 3%. Such inflation targeting is also prevalent throughout Europe. Such expectations are likely to affect price setting, wage bargaining and asset allocation, for example. For monetary authorities, therefore, it is important to know what drives inflationary expectations. Very little work, appears to have been undertaken which seeks to explain inflationary expectations at an individual level. Here we use Australian unit record data from the Melbourne Institute's Survey of Consumer Inflationary Expectations. The survey is a stratified random sample of 1,200 respondents, conducted monthly. Respondents are asked a wide variety of questions, including what they expect inflation to be over the coming year. In addition, many personal demographics are also recorded. Three months (February, March and April, 1999) were pooled and treated as a single cross section. As official headline inflation in Australia, the Consumer Price Index, is a quarterly series, and the three months chosen span a stable inflationary climate (the annual rate of increase in the CPI Q1, 1999 was 1.2%), such a pooling appears to be justified.

In our analysis we adopt a framework within which it is assumed that in forming their inflationary expectations, individuals are utility maximizers. That is, the extent of "sophistication" of their forecasts (in terms of expenditures of time and resources in obtaining such) will be an increasing function of the benefits that they are likely to obtain from a more accurate forecast. Therefore, it is likely that, along with other (economic) variables, inflationary expectations are driven by certain socio-demographic attributes that the individual possesses. In other words, different individuals have access to different information sets. These different information sets are, in part, a function of personal characteristics or attributes. Moreover, the extent to which these information sets are used is, again in part, determined by the individual's utility maximizing process.

It is expected *a priori*, that the accumulation of knowledge with age is likely to exert a positive influence on the sophistication of individuals' forecasts, as is the level of education. Those individuals with a close proximity to the price setting process are likely to be more aware of the inflationary climate. In the empirical example, we choose occupations of "managers" and "sales-persons" to proxy these. An indicator of voting intentions is also included, as there is evidence that Opposition voters tend to have more pessimistic inflationary expectations (see Brischetto and DeBrouwer 1999). We also include a dummy for place of residence, as this also may affect available information sets. Finally, it has been argued that there will be differences between male and female inflationary expectations, to the extent that one gender spends relatively more time retail shopping (Batchelor and Jonung 1986).

In our data observed inflationary expectations contained some quite large values, ranging from -50% to 80%. Estimation was based upon only those respondents who elicited a "sensible" expectation, defined as $0\% \leq P^e \leq 10\%$ (Brischetto and DeBrouwer 1999).[3] Selecting the data in this way yields results that are not based on unrealistic observations (outliers) and yields a data set with an appropriate number of alternatives, which additionally are contiguous. Importantly, the data are ordered, discrete (only integer responses are recorded) and there is significant evidence of *digit preferencing* (expectations of 0, 5 and 10% are quite obviously favored). In addition to the digit preferencing, there is also significant mass at expectation points 2 and 3%, which correspond to the prevailing headline rate at the time of the surveys, and moreover the RBA's inflation target range of 2-3%. *A priori*, one would expect the digit preferenced outcomes to exhibit a non-zero extent of captivity.

4.1 Results

The parameter estimates for the DOGEV model are reported in Table 2. Once more, the model seems to be relatively well specified with strong significance on many of the observed demographic variables. Ordering appears to be important, as ρ is strongly significant. Indeed, ρ differs significantly from both zero and one. Since correlation is inversely related to ρ, we note that the estimated value of 0.5 implies a relatively high level of correlation between adjacent categories (expectations). Considering the captivity parameters we find significant parameters for expectations of 0%, 1%, 2% and 7% (θ_0, θ_1, θ_2 and θ_7). Interestingly, the largest effect is given by θ_2 which corresponds closely to the well publicized target inflation band of 2-3% per

[3] Of course, in a low inflation environment, some negative expected inflation rates may be considered reasonable.

annum. As expected the heavily digit-preferenced outcome of zero inflation, is significant, although somewhat surprisingly, not that for either 5 or 10%. Note that, unlike the previous example, some of the θ values have gone to their lower boundary solutions of zero (θ_5 and θ_9).

In summary, we can say that these results clearly indicate that both captivity and ordering effects should be in this model of inflationary expectations. The DOGEV model is, therefore, clearly an improvement over all of its nested sub-models.

5 Application 3: Student Evaluations

Our final application involves the analysis of students' evaluation of a large first year, compulsory introductory statistics subject, taught in a business faculty in an Australian University. The data comprises 1,647 students covering the four years 1997 to 2000 and interest focusses upon the response to one particular question "*Overall I was satisfied with the quality of this subject*". Students respond to this by indicating their strength of agreement on a five point scale from *strongly disagree* to *strongly agree*. It is not unusual for the average score on this question to be used as evidence in the tenure and promotion processes, and in inter-department or cross faculty comparisons. However, various levels of disquiet are typically expressed about this process as reliance on the single figure might blur the issues.

Here we take responses to certain other questions in the evaluation surveys to help understand the level of student satisfaction with the quality of the subject. The (explanatory) variables chosen are the responses - level of agreement - with the statements: "*The subject was intellectually stimulat-*

ing" (C4), "*The subject developed my skills in areas relevant to the field*" (C3), "*The subject developed my understanding of key concepts*" (C2) and "*The tutor is readily available for consultation with the student*" (T5). For parsimony these variables are treated as continuous explanatory variables in our modeling. Additionally, we also include indicator variables for the survey year.

The evaluation surveys do not contain any information on the individual students, such as class attendance, academic performance, age or gender. As a result we might expect the θ parameters to play an important role, as they may absorb some unobserved heterogeneity of the individual, as well as that of the choice outcome. Furthermore, *a priori*, we might expect there to be some attraction to the two ends of the scale that the responses to the statements C4, C3, C2 and T5 may not be able to capture. This would suggest that only these two end θ parameters may be significant. Alternatively, one could *a priori* restrict the appropriate θ parameters to zero. The DOGEV model allows us to investigate these issues.

5.1 Results

The parameter estimates for the DOGEV model are reported in Table 3. In preliminary estimations we found that the θ parameters relating to disagree, neutral and agree were indeed, statistically insignificant. Thus the model presented in Table 3 has excluded these parameters. Likelihood ratio tests of the DOGEV model against its nested sub-models MNL, DOGIT and OGEV were 45.96, 15.99, and 29.10 with 3, 1 and 2 degrees of freedom respectively. Clearly the DOGEV specification is to be preferred. Wald tests of $\rho = 1$ or

$\rho = 0$ also reject the null.

The results therefore, indicate clear evidence of ordering and of captivity (to the extremes of the choice outcome scale). However, the actual degree of captivity is small with the choice set (captivity) probabilities for strongly disagree being 0.005 and for strongly agree being 0.006. Hence, in practical terms, the responses to C4, C3, C2, T5 and the year effects determine the outcome. For example, in Table 4 we use the model to predict the probability of stylized students (in 1997) who are "*dissatisfied*" (strongly disagree to the statements C4, C3, C2 and T5), "*neutral*" (undecided/neutral to the statements) and "*satisfied*" (strongly agree to the statements). In addition, we also report the (full) sample proportions of observed choices. As can be seen, a *dissatisfied* student will have an 85% chance of overall being dissatisfied with the quality of the subject, this compares with 14% for the *neutral* student. At the other end of the spectrum, the *satisfied* student has a 98% chance of overall being satisfied with the quality of the subject. Using the maximum probability rule, we would predict that the *dissatisfied*, *neutral* and *satisfied* student, would respectively disagree, be neutral and strongly agree, to the statement "*Overall I was satisfied with the quality of this subject*". It is important to note that the θ parameters corresponding to the ends of the response scale, help in putting probability mass into these infrequently chosen alternatives.

6 Conclusions

In this paper we introduced a new model, the Dogit Ordered Generalized Extreme Value (DOGEV) model, that can deal with data that is both ordinal and potentially embodies an extent of captivity (which may, or may not, be reflected by a multi-modal distribution of observed outcomes). The DOGEV model is attractive with regard to three main facets: it has the flexibility of the multinomial logit (MNL) model in allowing coefficients to vary across alternatives (Small 1987), it embodies the ordering and correlation of proximate outcomes properties of the ordered generalized extreme value (OGEV) model and it also allows one to estimate the extent of any captivity/preferencing/loyalty in the choice process, as exhibited by the DOGIT model. It is a simple, parsimonious and flexible model that has the additional benefit of nesting certain key sub-models (DOGIT, OGEV and MNL).

The attractiveness and potential usefulness of this new model was clearly illustrated with three separate applications. Firstly, it was applied to an employment contract-type model, where there was some evidence of ordering and compelling evidence that workers are captive (to differing extents) to the various contract types, to an extent over and above that suggested simply by their personal characteristics. There was stronger evidence of ordering with regard to inflationary expectations, although somewhat surprisingly, individuals did not appear to be particularly captive to the *a priori* digit preferenced numbers/outcomes. Finally, there was again compelling evidence for ordering in our student evaluation data. Here however, we restrict some captivity parameters to be zero *a priori* which helps to put probability mass into the infrequently chosen outcomes, on the basis of a paucity of observed individual characteristics. Indeed, these (extreme) captivity parameters were strongly significant.

In summary, the applications have shown that the model is likely to be attractive in situations where one wishes to model unit record data that are both ordinal and potentially exhibit an extent of captivity in the choice outcome process. Moreover, it appears straightforward to extend the model to explain the captivity parameters (which may reflect demand-side effects, for example) in terms of observables - thus giving much greater insight into the economic data generating process than is afforded by more simple discrete choice models. Indeed, it is likely that such models will be applicable to many areas of interest, such as occupational choice and discretized labor supply choice.

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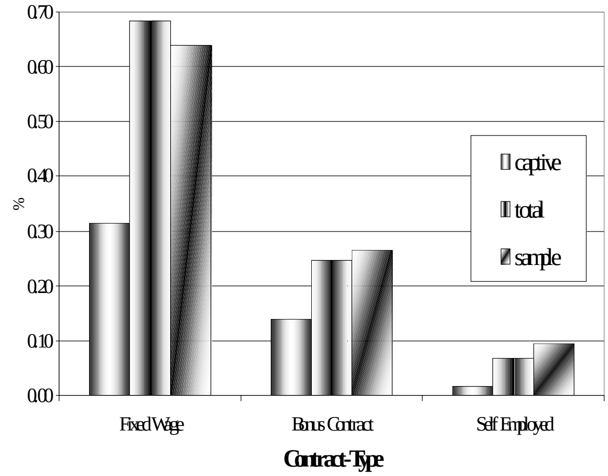

Figure 1: Implied DOGEV Probabilities and Sample Proportions

Table 1: Employment Contract Types

	PRP		Self Employed	
Constant	-4.02	(1.31)**	-8.37	(1.39)**
Black	-0.43	(0.44)	-0.38	(0.44)
Asian	-1.29	(0.84)	0.43	(0.27)
Other Ethnic	0.01	(0.40)	0.23	(0.44)
Male	0.61	(0.21)**	1.20	(0.25)**
Married	0.16	(0.13)	0.59	(0.21)**
Separated	0.24	(0.20)	0.60	(0.26)**
Wales	-0.41	(0.30)	-0.09	(0.22)
Scotland	-0.17	(0.30)	-0.07	(0.29)
North	0.04	(0.15)	-0.12	(0.16)
Midlands	-0.27	(0.17)	-0.33	(0.17)*
South	0.10	(0.12)	-0.30	(0.13)**
G.C.S.E.s	0.16	(0.16)	-0.06	(0.12)
Further Education	0.35	(0.20)*	-0.04	(0.15)
Higher Education	0.16	(0.17)	-0.07	(0.15)
Renting	0.03	(0.25)	1.32	(0.30)**
Mortgage	0.71	(0.30)**	0.80	(0.22)**
House Owned	0.38	(0.25)	1.19	(0.27)**
Managerial	0.04	(0.15)	-0.52	(0.18)**
Skilled	-0.70	(0.33)**	0.05	(0.17)
Semi Skilled	-0.71	(0.33)**	-0.55	(0.25)**
Unskilled	-1.22	(0.56)**	0.02	(0.31)
Manufacturing	-0.01	(0.11)	-0.71	(0.17)**
Services	-1.51	(0.45)**	-1.58	(0.31)**
Age/10	1.69	(0.66)**	2.38	(0.53)**
Age2/100	-0.23	(0.09)**	-0.23	(0.06)**
# Pre-school Children	0.15	(0.11)	0.36	(0.13)**
# Children aged 5-18	-0.24	(0.10)**	0.18	(0.06)**
$\theta_{salaried}$	0.59	(0.30)**		
θ_{PRP}	0.26	(0.08)**		
$\theta_{self-employed}$	0.03	(0.01)**		
ρ	0.80	(0.37)**		
N	13,675			
logL	-11,060			

Table 2: Inflationary Expectations Results

	$j=1$		$j=2$		$j=3$		$j=4$		$j=5$	
Constant	-1.73	(1.92)	0.17	(1.08)	1.03	(1.01)	0.32	(1.45)	2.23	(0.59)**
Male	0.11	(0.52)	-0.09	(0.42)	-0.32	(0.07)**	-0.36	(0.23)	-0.80	(0.20)**
Age	-0.06	(0.10)	-0.23	(0.17)	-0.08	(0.05)	-0.07	(0.06)	-0.09	(0.05)*
Income	-0.12	(0.06)**	-0.09	(0.07)	-0.10	(0.05)**	-0.12	(0.05)**	-0.13	(0.04)**
Urban	0.59	(0.67)	-0.48	(0.48)	-0.09	(0.19)	-0.14	(0.21)	0.06	(0.17)
Manager	0.39	(0.61)	0.79	(0.83)	-0.17	(0.22)	-0.30	(0.26)	-0.61	(0.23)**
Sales	-1.44	(1.67)	0.55	(0.82)	-0.27	(0.29)	-0.29	(0.31)	-0.47	(0.26)*
Education	0.11	(0.10)	-0.02	(0.09)	0.01	(0.04)	0.07	(0.09)	-0.02	(0.04)
Opposition	0.51	(0.37)	0.81	(0.47)	0.58	(0.20)**	0.93	(0.29)**	0.91	(0.28)**
	$j=6$		$j=7$		$j=8$		$j=9$		$j=10$	
Constant	-3.80	(6.75)	-2.62	(1.59)*	-1.95	(1.56)	0.53	(1.20)	2.04	(0.63)**
Male	0.95	(4.81)	-0.66	(0.23)**	-0.36	(0.43)	-0.20	(0.69)	-1.09	(0.26)**
Age	0.25	(0.42)	0.24	(0.20)	0.11	(0.15)	-0.48	(0.25)**	-0.11	(0.06)**
Income	-0.14	(0.14)	-0.39	(0.23)*	-0.09	(0.09)	-0.09	(0.13)	-0.19	(0.06)**
Urban	0.31	(0.62)	1.07	(0.04)**	0.14	(0.46)	-1.75	(0.88)**	-0.19	(0.19)
Manager	0.31	(0.92)	0.51	(0.18)**	-0.10	(0.56)	0.01	(1.64)	-0.39	(0.28)
Sales	1.03	(2.12)	1.59	(0.85)*	-0.48	(0.81)	-2.88	(4.65)	-0.22	(0.29)
Education	-0.27	(0.17)	-0.50	(0.52)	-0.02	(0.10)	-0.11	(0.19)	-0.03	(0.05)
Opposition	1.84	(0.34)**	2.35	(0.87)**	1.15	(0.48)**	1.00	(0.66)	1.42	(0.32)**
θ_0	0.05	(0.03)*								
θ_1	0.02	(0.01)**								
θ_2	0.17	(0.03)**								
θ_3	0.02	(0.20)								
θ_4	0.01	(0.07)								
θ_5	0.00	-								
θ_6	0.00	(0.01)								
θ_7	0.02	(0.00)**								
θ_8	0.01	(0.01)								
θ_9	0.00	-								
θ_{10}	0.01	(0.05)								
ρ	0.50	(0.20)**								
N	2,194									
logL	-4,197									

Table 3: Student Evaluations

	Disagree		Neutral		Agree		Strongly Agree	
Constant	-1.32	(1.19)	-3.13	(1.24)**	-7.64	(1.33)**	-19.96	(2.27)**
C4	0.89	(0.33)**	1.18	(0.34)**	1.52	(0.36)**	2.04	(0.37)**
C3	0.69	(0.25)**	0.89	(0.26)**	1.30	(0.27)**	2.10	(0.37)**
C2	0.17	(0.22)	0.51	(0.23)**	0.89	(0.23)**	1.80	(0.32)**
T5	-0.07	(0.11)	0.01	(0.11)	0.29	(0.12)**	0.80	(0.24)**
1998	-0.72	(0.97)	-1.50	(1.00)	-1.86	(1.02)*	-0.78	(1.09)
1999	-1.16	(0.83)	-1.33	(0.83)	-1.65	(0.84)*	-1.61	(0.86)*
2000	-0.95	(0.87)	-1.08	(0.88)	-1.23	(0.89)	-1.37	(0.92)
$\theta_{strongly\ disagree}$	0.01	(0.00)**						
$\theta_{strongly\ agree}$	0.01	(0.00)*						
ρ	0.26	(0.08)**						
N	1,635							
logL	-1,610							

Table 4: Student Evaluations: Predicted Probabilities for Stylized Students

Overall I was satisfied with the quality of this subject:	Stylized Student Type: Dissatisfied	Neutral	Satisfied	Sample
Strongly Disagree	0.29	0.01	0.01	0.04
Disagree	0.56	0.13	0.00	0.21
Neutral	0.13	0.54	0.01	0.28
Agree	0.01	0.31	0.11	0.40
Strongly Agree	0.01	0.01	0.87	0.07